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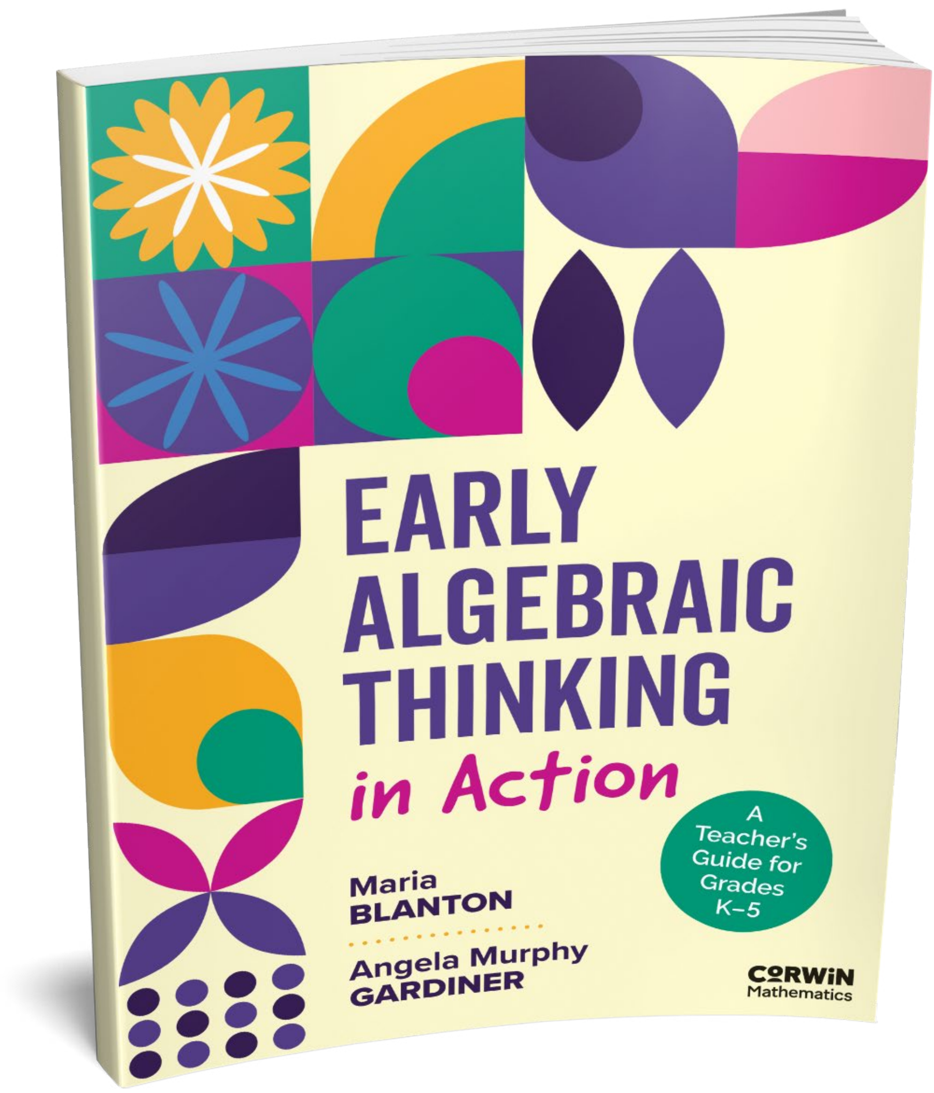

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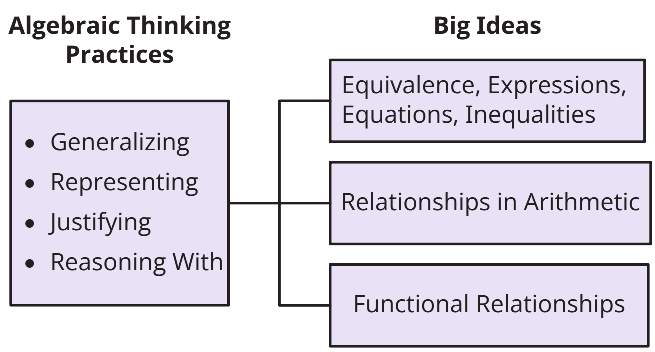
WHAT IS EARLY ALGEBRA?

Introducing algebra as early as kindergarten (e.g., NGA Center & CCSSO, 2010) raises a fair question: "*What* algebra?" What is the algebra we want young children to learn? How would you define algebra? If you think of algebra as largely about manipulating variables to simplify difficult expressions or solve complicated equations, you might rightly question whether young children should spend their time on this in math class. We agree! Algebra in the elementary grades—that is, *early algebra*—is not "algebra early" (Carraher et al., 2008). It isn't about repackaging all the procedures and processes you might have learned in an algebra class for young children.

NCTM's *Principles and Standards* (NCTM, 2000) states that students who "reason and think analytically tend to note patterns, structure, or regularities" (p. 56). This captures the intent of early algebra: developing mathematical thinkers who notice, understand, and can reason with mathematical structure and relationships. We frame early algebra as a set of four key algebraic thinking practices: generalizing, representing, justifying, and reasoning with mathematical structure and relationships (Blanton, Brizuela et al., 2018). In the elementary grades, these practices occur primarily in several core content areas that we call "Big Ideas" (see Figure 1.1). In this chapter, we briefly introduce the four algebraic thinking practices and describe the Big Ideas. We will look at these more closely in Chapters 2–4.

Figure 1.1 • *Algebraic thinking practices and the Big Ideas where they can occur*



Generalizing Relationships

The practice of *generalizing* is so important that it has been described as the heart of algebraic thinking (Cooper & Warren, 2011). When students generalize, they notice a general mathematical relationship that *might be true across some domain or set of numbers*. In other words, they see the underlying relationship that connects a set of specific cases in a particular way. For example, they don't simply see that $5 + 4$ is odd. They notice that the sum of *any* odd number and *any* even number is *always* odd. They don't just see that 3×12 is the same as 12×3 because the order of the factors is switched. They recognize that the order in which they multiply *any* two numbers does not change their product. They might notice that the number of bicycle wheels is always double the number of bicycles, no matter the number of bicycles.

Generalizations are the building blocks of thinking algebraically. When students notice general relationships, they are looking for and making use of structure, an important *Common Core* mathematical practice (NGA Center & CCSSO, 2010). When their generalizations arise from "real world" contexts, like functional relationships, they are engaging in another important mathematical practice—modeling with mathematics (NGA Center & CCSSO, 2010). Students of all ages should be given opportunities to generalize.

TIPS

KEY IDEA: *Generalizing* is the heart of algebraic thinking. Students are generalizing when they notice mathematical structure or general relationships.

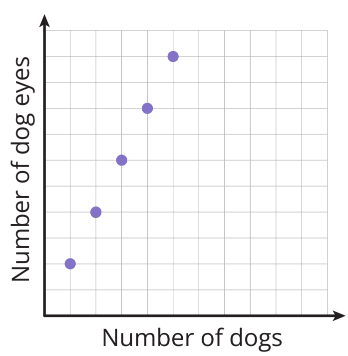
Representing Relationships

Once students notice a relationship, they can *make a conjecture* to describe what they see. The *Principles and Standards* (NCTM, 2000) is clear: Students should "make and investigate mathematical conjectures" (p. 57). Conjecturing is a way to introduce a general claim, or *generalization*, into classroom conversations. Students usually do this by stating a conjecture in their own words. Using natural language allows them to express generalizations in ways that make sense to them. For example, as a child explores subtraction, they might observe, "I noticed that any time I take away a number from itself, I get zero."

Students can represent relationships in a variety of ways. In addition to their own words, students might use tables, graphs, pictures, or even variable notation. They might represent the claim, "If I subtract a number from itself, I get zero" with variable notation as $a - a = 0$, for any number a. They might describe the relationship between the number of dogs and number of dog eyes in words as "The number of dog eyes is double the number of dogs." They might represent this relationship using variable notation as $n = d + d$ or $n = 2 \times d$, where d represents the number of dogs and n the number of dog eyes. They might also represent this relationship in a table or with a coordinate graph (see Figure 1.2).

Figure 1.2 • *A table and graph representing the "dog eyes" relationship*

Number of Dogs	Number of Dog Eyes
1	2
2	4
3	6
4	8



Students who can represent relationships in different ways and who understand how these representations are connected have a richer understanding of the relationships (Brizuela & Earnest, 2008). For example, they understand that the way the number of dog eyes increases in the table will result in the graph of a line that shows "steady" growth (see Figure 1.2). Encouraging students to use different representations in this way and to think about how these representations relate to each other is important for developing *representational fluency*.

TIPS

KEY IDEA: Students can represent mathematical relationships in different ways using words, variable notation, tables, pictures, and graphs. *Representational fluency* means understanding different ways to represent mathematical relationships and how these representations are connected.

Variable notation is the representation that might seem the most out of place in elementary grades. Many people think using letters to represent mathematical quantities is only for older students. Given the challenges older students are known to have with variable notation, wouldn't younger children have even more difficulties? After almost two decades of research on variable notation in elementary classrooms, we have found the opposite to be true (e.g., Blanton et al., 2017; Blanton, Stephens et al., 2015)! *As elementary students explore and use variable notation, they develop meaning for it and can use it to represent mathematical relationships.* You might be surprised to learn that elementary students sometimes use variable notation more easily than their own words (Blanton et al., 2019)!

If this seems counterintuitive, think about it this way: How would children make sense of highly complex language in written form if they were not introduced to the "squiggles" that they come to know as letters at an early age? For example, a child has no prior knowledge of the marks that form the letter *A.* Nor is there anything inherent in the shape of the marks that shows what "A" means. Yet parents start introducing letters to their children long before formal schooling begins. Why? Because as children interact with letters in context (for example, seeing the letter "A" associated with a picture of an apple and hearing the short A sound the letter makes), they come to develop meaning for this symbol. Imagine how children would fare in their understanding of written language if they were not introduced to letters until middle grades! The challenge of developing meaning for letters, understanding how to combine letters to form words, and finally, combining words to form sentences using an (arbitrary) system of rules, would be huge. In the same way, variable notation is an important tool for students to learn to use in elementary grades, long before formal algebra.

TIPS

TAKE A LOOK: What kinds of representations does your curriculum ask students to use? What are students being asked to represent? Are students asked to use words, variable notation, tables, drawing, or graphs to represent general claims?

Justifying Relationships

Generalizing and representing relationships are not the only ways children can think algebraically. When students make a conjecture about a mathematical relationship, they are making a claim about what they think *might be true*. In this way, a conjecture acts as a "major pathway to discovery" (NCTM, 2000, p. 57). Knowing whether a conjectured relationship is true is just as important as making the claim.

TIPS

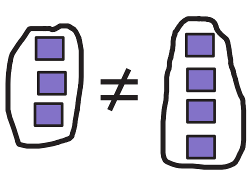
KEY IDEA: When students make a conjecture about a mathematical relationship, they are making a claim about what they think *might be true*.

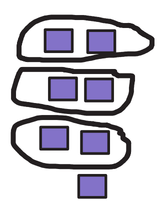
TIPS

THINK ABOUT IT: Students sometimes make general claims that aren't true. For example, they sometimes say that multiplying any two numbers always makes a "bigger" number. Why is this claim false?
What is a generalization that your students have made that is not true? How did you address this with your students?

Building viable arguments to show a conjecture is true is an important mathematical practice (NGA Center & CCSSO, 2010). From the early elementary grades, students should experience mathematics as a sense-making activity in which understanding *why* things work is central to *doing* mathematics. Without this as part of their early learning experiences, students will likely have more difficulties with mathematical content in later grades (Knuth et al., 2002; Stylianides & Stylianides, 2008).

Some arguments are meant to justify claims made about specific cases. For example, students might be asked to explain why 3 + 4 is an odd number. They might argue that 7 is an odd number because 7 cubes cannot be separated into pairs of cubes or into two equal groups:





These arguments are important to make, but they are different than an argument that shows a *general* claim is true. Arguments showing a general relationship is true must show it's true for *all* cases on which the claim is made. But many of the relationships that students notice are claims made for an infinite set of numbers. For example, the conjecture “any number subtracted from itself is zero” is a claim made for all real numbers. How is it possible to show a conjecture is true for every single case when there are an infinite number of cases? What makes a *good* argument for why a mathematical relationship is *always* true?

Students often use *empirical arguments* to show a claim is true. Empirical arguments are based on testing whether some (not all) examples of a conjectured relationship are true or false. For example, students might test several cases of the conjecture “the sum of any whole number plus 2 is even.” Suppose they test this claim by adding 2 to the whole numbers 4, 8, and 10: $4 + 2$, $8 + 2$, and $10 + 2$. Since the sum of each of these is even, they might decide that the conjecture is true. Students find this type of argument very convincing. But is an empirical argument a good one? In this illustration, students might have randomly picked 4, 8, and 10, not realizing that their choice of numbers led them to think the conjecture is true when it is not. If they had chosen an odd number, such as 7, they would see that the conjecture is not true since $7 + 2$ is not even.

A mathematical argument can be convincing to students but still not a very good one. To be clear, looking at different cases or examples of a mathematical claim can help students make sense of the claim (Knuth et al., 2019). For example, if students don’t know the answer to the question “Is the sum of two odd numbers even or odd?,” then looking at different cases of adding two odd numbers would be a good way to figure this out. They might then make a conjecture, such as “If two odd numbers are added, their sum is aways even.” But an argument that only tests some cases of a claim is not a good mathematical argument because it doesn’t show the relationship is true *for all cases*. (Keep in mind that a good mathematical argument is different from arguments students make in science class! In science, arguments are often empirical. Hypotheses are tested through observations, and conclusions are made by considering “cases.”)

TIPS

KEY IDEA: A good mathematical argument must show that a conjecture is true *for all* values for which the claim is made. Arguments that just look at some specific cases of a claim might be convincing, but they are not good mathematical arguments.

What options do young learners have for building good arguments? They don’t yet know the methods older students might use to build formal arguments, yet testing examples is not the basis of a good argument. It turns out, they already have tools on hand and ways of reasoning they can use to build good arguments! As early as kindergarten, children can start to build *representation-based arguments* (Schifter, 2009) to show relationships are true. As we explore later in this book, representation-based arguments do not rely on testing examples. They use diagrams, manipulatives, and story contexts—tools accessible to young children—to show why a conjecture is true *for all cases* on which it is made. Building good arguments for why a general claim is true is important mathematical thinking.

TIPS

TAKE A LOOK: What kinds of mathematical claims does your curriculum ask students to make? Does it ask students to make general claims for a class of numbers? Does it ask students to build arguments to support their claims? If so, what kinds of arguments?

Reasoning With Relationships

Reasoning with general relationships is another important way to think algebraically. Ultimately, we want students to be able to use the relationships they notice to solve new math problems. Using structure in this way is an important mathematical practice (NGA Center & CCSSO, 2010). For example, if students know that the sum of two even numbers is even, they might use this to think about whether the sum of three even numbers is even or odd. Another way to reason with relationships is to use properties of operations—even intuitively—to make computational work more efficient (Blanton et al., 2011). For example, a student might compute $4 + 8 - 4 - 8$ by first finding $4 - 4$ and $8 - 8$. Even if the child can't explicitly describe their actions, they are reasoning with properties such as the Commutative Property of Addition and the Additive Inverse Property. This shows a much deeper understanding of arithmetic than simply computing left to right.

TIPS

KEY IDEA: Children are thinking algebraically when they reason with established relationships, such as arithmetic or functional relationships, in new problem situations or in computational work.

NCTM's *Principles and Standards* (NCTM, 2000) states that making sense of mathematics is a process of "developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas" (p. 56). Certainly, this includes generalizing, representing, justifying, and reasoning with mathematical structure and relationships! These practices are essential for algebraic (*and* arithmetic) thinking. And they help students make sense of mathematics. We will explore these practices throughout the remainder of this book, so keep their meanings in mind.